

Answers to Problem Set 10

5 marks each, total 45 marks

12.30

$\exists x (\neg \text{Cube}(x) \wedge \neg \text{Dodec}(x))$
 $\exists x \forall y \text{ SameShape}(x, y)$
 $\forall x \text{ Tet}(x)$

Shape Axioms.

- | | |
|--|--|
| 1. $\neg \exists x (\text{Cube}(x) \wedge \text{Tet}(x))$ | 6. $\forall x \forall y ((\text{Dodec}(x) \wedge \text{Dodec}(y)) \rightarrow \text{SameShape}(x, y))$ |
| 2. $\neg \exists x (\text{Tet}(x) \wedge \text{Dodec}(x))$ | 7. $\forall x \forall y ((\text{Tet}(x) \wedge \text{Tet}(y)) \rightarrow \text{SameShape}(x, y))$ |
| 3. $\neg \exists x (\text{Dodec}(x) \wedge \text{Cube}(x))$ | 8. $\forall x \forall y ((\text{SameShape}(x, y) \wedge \text{Cube}(x)) \rightarrow \text{Cube}(y))$ |
| 4. $\forall x (\text{Tet}(x) \vee \text{Dodec}(x) \vee \text{Cube}(x))$ | 9. $\forall x \forall y ((\text{SameShape}(x, y) \wedge \text{Dodec}(x)) \rightarrow \text{Dodec}(y))$ |
| 5. $\forall x \forall y ((\text{Cube}(x) \wedge \text{Cube}(y)) \rightarrow \text{SameShape}(x, y))$ | 10. $\forall x \forall y ((\text{SameShape}(x, y) \wedge \text{Tet}(x)) \rightarrow \text{Tet}(y))$ |

Proof:

From P1 we know that some object exists that is neither a cube nor a dodec. Let c be such an object. Then, since from **A4** we know that c is either a tet, dodec or cube, it's clear that $\text{Tet}(c)$. Further, from P2 we see that there's one object that's the same shape as every object in the domain. In other words, everything in the world has the same shape. Now, to prove that everything is a tet, let e be an arbitrary object in the domain. From P2 we know that c and e are the same shape. Now, since we know $\text{Tet}(c)$ we can use **A10** to get $\text{Tet}(e)$. Now, e was an arbitrary object, so from $\text{Tet}(e)$ we can infer $\forall x \text{ Tet}(x)$ by $\forall\text{Intro}$. ■

[N.B. A formal proof is, technically, a special case of an informal proof, and so *full marks should be given for a (correct) formal proof*.

13.3

1. $\forall x \text{ Cube}(x)$
2. $\forall x \text{ Small}(x)$
3. \boxed{a} ▾
4. $\text{Cube}(a)$ ✓ ▾ $\forall \text{ Elim: } 1$
5. $\text{Small}(a)$ ✓ ▾ $\forall \text{ Elim: } 2$
6. $\text{Cube}(a) \wedge \text{Small}(a)$ ✓ ▾ $\wedge \text{ Intro: } 5,4$
7. $\forall x (\text{Cube}(x) \wedge \text{Small}(x))$ ✓ ▾ $\forall \text{ Intro: } 3-6$

13.8

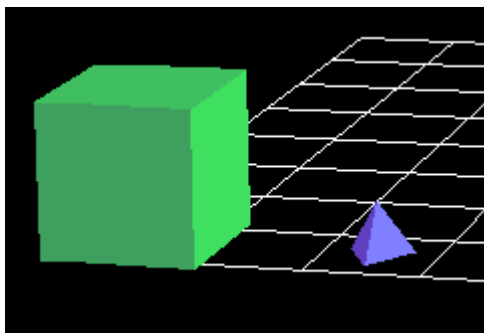
1. $\forall x (\text{Cube}(x) \rightarrow \forall y (\text{Dodec}(y) \rightarrow \text{FrontOf}(x, y)))$
2. \boxed{a} ▾
3. \boxed{b} ▾
4. $\text{Cube}(a) \wedge \text{Dodec}(b)$
5. $\text{Cube}(a) \rightarrow \forall y (\text{Dodec}(y) \rightarrow \text{FrontOf}(a, y))$ ✓ ▾ $\forall \text{ Elim: } 1$
6. $\text{Cube}(a)$ ✓ ▾ $\wedge \text{ Elim: } 4$
7. $\forall y (\text{Dodec}(y) \rightarrow \text{FrontOf}(a, y))$ ✓ ▾ $\rightarrow \text{ Elim: } 5,6$
8. $\text{Dodec}(b) \rightarrow \text{FrontOf}(a, b)$ ✓ ▾ $\forall \text{ Elim: } 7$
9. $\text{Dodec}(b)$ ✓ ▾ $\wedge \text{ Elim: } 4$
10. $\text{FrontOf}(a, b)$ ✓ ▾ $\rightarrow \text{ Elim: } 8,9$
11. $(\text{Cube}(a) \wedge \text{Dodec}(b)) \rightarrow \text{FrontOf}(a, b)$ ✓ ▾ $\rightarrow \text{ Intro: } 4-10$
12. $\forall y ((\text{Cube}(a) \wedge \text{Dodec}(y)) \rightarrow \text{FrontOf}(a, y))$ ✓ ▾ $\forall \text{ Intro: } 3-11$
13. $\forall x \forall y ((\text{Cube}(x) \wedge \text{Dodec}(y)) \rightarrow \text{FrontOf}(x, y))$ ✓ ▾ $\forall \text{ Intro: } 2-12$

N.B. In cases like this, I told students that they could introduce the double universal in one step. Thus steps 2 and 3, and 12 and 13, may be combined. They can even introduce both universals and the conditional in one step, combining 2, 3 and 4 in one line, and 11, 12 and 13.

13.13

- | | |
|---|---|
| 1. $\forall y (Cube(y) \vee Dodec(y))$ | |
| 2. $\forall x (Cube(x) \rightarrow Large(x))$ | |
| 3. $\exists x \neg Large(x)$ | |
| 4. $\boxed{c} \neg Large(c)$ | |
| 5. $Cube(c) \rightarrow Large(c)$ | ✓ ▾ \forall Elim: 2 |
| 6. $\neg Cube(c)$ | ✓ ▾ MT: 4,5 |
| 7. $Cube(c) \vee Dodec(c)$ | ✓ ▾ \forall Elim: 1 |
| 8. $Dodec(c)$ | ✓ ▾ DS: 6,7 |
| 9. $\exists x Dodec(x)$ | ✓ ▾ \exists Intro: 8 |
| 10. $\exists x Dodec(x)$ | ✓ ▾ \exists Elim: 4-9,3 |

13.25



T 1. $\exists x Cube(x) \wedge \exists x Small(x)$

F 2. $\exists x (Cube(x) \wedge Small(x))$

13.33

1. $\neg \exists x (Tet(x) \wedge Small(x))$	
2. $\forall y (Small(y) \vee Medium(y) \vee Large(y))$	
3. $\boxed{a} \quad \nabla \quad Tet(a)$	
4. $\nabla \quad Small(a)$	
5. $Tet(a) \wedge Small(a)$	✓ $\nabla \wedge$ Intro: 4,3
6. $\exists x (Tet(x) \wedge Small(x))$	✓ $\nabla \exists$ Intro: 5
7. \perp	✓ $\nabla \perp$ Intro: 1,6
8. $\neg Small(a)$	✓ $\nabla \neg$ Intro: 4-7
9. $Small(a) \vee Medium(a) \vee Large(a)$	✓ $\nabla \vee$ Elim: 2
10. $Medium(a) \vee Large(a)$	✓ ∇ DS: 9,8
11. $\nabla \quad Medium(a)$	
12. $Large(a) \vee Medium(a)$	✓ $\nabla \vee$ Intro: 11
13. $\nabla \quad Large(a)$	
14. $Large(a) \vee Medium(a)$	✓ $\nabla \vee$ Intro: 13
15. $Large(a) \vee Medium(a)$	✓ $\nabla \vee$ Elim: 11-12,13-14,10
16. $\forall x (Tet(x) \rightarrow (Large(x) \vee Medium(x)))$	✓ $\nabla \forall$ Intro: 3-15

13.37

1. $\forall x (Dodec(x) \rightarrow LeftOf(x, a))$	
2. $\forall x (Tet(x) \rightarrow RightOf(x, a))$	
3. $\forall x \forall y (LeftOf(x, y) \rightarrow \neg SameCol(x, y))$	
4. $\forall x \forall y (RightOf(x, y) \rightarrow \neg SameCol(x, y))$	
5. $\forall x (Cube(x) \vee Dodec(x) \vee Tet(x))$	
6.  \triangledown SameCol(c, a)	
7. $Cube(c) \vee Dodec(c) \vee Tet(c)$	✓ \triangledown \vee Elim: 5
8. $Dodec(c) \rightarrow LeftOf(c, a)$	✓ \triangledown \vee Elim: 1
9. $Tet(c) \rightarrow RightOf(c, a)$	✓ \triangledown \vee Elim: 2
10. $\forall y (LeftOf(c, y) \rightarrow \neg SameCol(c, y))$	✓ \triangledown \vee Elim: 3
11. $LeftOf(c, a) \rightarrow \neg SameCol(c, a)$	✓ \triangledown \vee Elim: 10
12. $\forall y (RightOf(c, y) \rightarrow \neg SameCol(c, y))$	✓ \triangledown \vee Elim: 4
13. $RightOf(c, a) \rightarrow \neg SameCol(c, a)$	✓ \triangledown \vee Elim: 12
14. \triangledown Cube(c)	
15. Cube(c)	✓ \triangledown Reit: 14
16. \triangledown Dodec(c)	
17. LeftOf(c, a)	✓ \triangledown \rightarrow Elim: 16,8
18. $\neg SameCol(c, a)$	✓ \triangledown \rightarrow Elim: 11,17
19. \perp	✓ \triangledown \perp Intro: 18,6
20. Cube(c)	✓ \triangledown \perp Elim: 19
21. \triangledown Tet(c)	
22. RightOf(c, a)	✓ \triangledown \rightarrow Elim: 21,9
23. $\neg SameCol(c, a)$	✓ \triangledown \rightarrow Elim: 22,13
24. \perp	✓ \triangledown \perp Intro: 6,23
25. Cube(c)	✓ \triangledown \perp Elim: 24
26. Cube(c)	✓ \triangledown \vee Elim: 7,14-15,16-20,21-25
27. $\forall x (SameCol(x, a) \rightarrow Cube(x))$	✓ \triangledown \forall Intro: 6-26

1.

1. $\forall x ((\text{Cube}(x) \wedge \exists y (\text{Cube}(y) \wedge x \neq y)) \rightarrow \text{Large}(x))$	
2. $\text{Cube}(a) \wedge \text{Cube}(b)$	
3. $\text{SameCol}(a, c) \wedge \neg \text{SameCol}(b, c)$	
4. $(\text{Cube}(a) \wedge \exists y (\text{Cube}(y) \wedge a \neq y)) \rightarrow \text{Large}(a)$	✓ ▾ \forall Elim: 1
5. $\text{Cube}(a)$	✓ ▾ \wedge Elim: 2
6. ▾ $a = b$	
7. $\text{SameCol}(a, c)$	✓ ▾ \wedge Elim: 3
8. $\text{SameCol}(b, c)$	✓ ▾ $=$ Elim: 7,6
9. $\neg \text{SameCol}(b, c)$	✓ ▾ \wedge Elim: 3
10. \perp	✓ ▾ \perp Intro: 8,9
11. $a \neq b$	✓ ▾ \neg Intro: 6-10
12. $\text{Cube}(b)$	✓ ▾ \wedge Elim: 2
13. $\text{Cube}(b) \wedge a \neq b$	✓ ▾ \wedge Intro: 12,11
14. $\exists y (\text{Cube}(y) \wedge a \neq y)$	✓ ▾ \exists Intro: 13
15. $\text{Cube}(a) \wedge \exists y (\text{Cube}(y) \wedge a \neq y)$	✓ ▾ \wedge Intro: 14,5
16. $\text{Large}(a)$	✓ ▾ \rightarrow Elim: 4,15

2.

1. $\forall x (\exists y \text{ Loves}(x, y) \rightarrow \text{Loves}(x, \text{celine}))$	
2. $\text{Loves}(\text{celine}, \text{bill})$	
3. $\neg \text{Loves}(\text{alice}, \text{alice})$	
4. ▾ $\text{alice} = \text{celine}$	
5. $\exists y \text{ Loves}(\text{celine}, y) \rightarrow \text{Loves}(\text{celine}, \text{celine})$	✓ ▾ \forall Elim: 1
6. $\exists y \text{ Loves}(\text{celine}, y)$	✓ ▾ \exists Intro: 2
7. $\text{Loves}(\text{celine}, \text{celine})$	✓ ▾ \rightarrow Elim: 5,6
8. $\neg \text{Loves}(\text{celine}, \text{celine})$	✓ ▾ $=$ Elim: 3,4
9. \perp	✓ ▾ \perp Intro: 7,8
10. $\text{alice} \neq \text{celine}$	✓ ▾ \neg Intro: 4-9